**Assignment 02**

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1. Q1. Prove that if ⊂ R ( R is set of real numbers) and S = { : i ≥ 0 } then ∀∈ S, ≠ φ if ⊂, > .

Answer:

* Let's denote = X, where is the intersection of all sets in S.
* Suppose ⊂, for some >
* Then, X ⊂ because X is the intersection of all and ⊂.
* Since X ⊂ X is a subset of each for >
* By the definition of intersection, X contains all elements that are common to each set for > .
* Since X is a subset of each for > , X itself contains these common elements.
* Thus, is not empty, as it contains at least one element that is common to all sets for >

Therefor, if ⊂ R and S = { : i ≥ 0 }, then for all in not empty if ⊂ and > .

Example:

Explanation of Notation:

* ∀ represents "for all" or "for every".
* denotes a set in the sequence of sets.
* S represents the set containing all sets
* denotes the intersection of sets where belongs to the index set .

Let's consider the sequence of sets:

= {1,2,3}

= {2,3}

= {3}

Here ⊃ ⊃ , as each subsequent set is a subset of the previous one.

Proof:

We want to show that for any set in the sequence, the intersection is non-empty.

Consider from our example. Since ⊃ ⊃ , we can take = {0,1,2}.

= ∩ ∩

= {1,2,3} ∩ {2,3} ∩ {3}

= {3}

As we can see, the intersection is not empty ({3} in this case).

Hence, for any sequence of sets ⊂ R such that ⊂ for some > , for any in S, the intersection is non-empty.

1. **Q2.** Prove that a surjective function does not have a preimage in the traditional sense.

**Answer:**

A surjective function is one for which every element in the codomain has at least one corresponding element in the domain. In other words, every element in the codomain is mapped to by at least one element in the domain.

To prove that a surjective function does not necessarily have a unique preimage for each element in the codomain, let's consider an example:

Consider the function *f*:R→R defined by *f*(x) = .

This function is surjective because for any y in the codomain (R), there exists an x in the domain (R) such that *f*(x) = y. This is because every real number has a square root in the real numbers.

However, if we take y = 4, for instance, there are two preimages: x = 2 and x = −2. So, = {2, −2}. This shows that a surjective function does not have a unique preimage for every element in the codomain.

Therefore, a surjective function does not necessarily have a unique preimage in the traditional sense.

1. **Q3.** Prove that if E ⊂ A\*B and *f* : E→R such that *f*((a,b)) = a+b then *f* may not have an inverse. Here R is a set of real numbers.

**Answer:**

To prove that the function *f*:E→R defined by *f*((a,b))=a+b may not have an inverse, we'll show that *f* is not injective (one-to-one).

Suppose we have two distinct elements (, ) and (, ) in E such that *f*((, )) = *f*((, )). According to the given function definition:

*f*((, )) = +

*f*((, )) = +

If *f* has an inverse, then by definition, = (, ) and = (, ).

However, if = , it doesn't necessarily imply that (, ) = (, ).

For example, consider = 1, = 2, = 3, and = 0.

Then:

*f*((1,2)) = 1+2 = 3

*f*((3,0)) = 3+0 = 3

Even though *f*((1,2)) = *f*((3,0)), (1,2) is not equal to (3,0).

Hence, *f* is not injective, and thus, it may not have an inverse.

This demonstrates that the function f((a,b))=a+b defined on E may not have an inverse due to its lack of injectivity.